

6.6 Videos Guide

6.6a

- A brief review of inverse trigonometric functions

6.6b

- Calculus of inverse trigonometric functions

$$\circ \frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \quad \Rightarrow \quad \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\circ \frac{d}{dx}(\cos^{-1} x) = \frac{1}{-\sqrt{1-x^2}}$$

$$\circ \frac{d}{dx}(\tan^{-1} x) = \frac{1}{x^2+1} \quad \Rightarrow \quad \int \frac{1}{x^2+1} dx = \tan^{-1} x + C$$

$$\circ \frac{d}{dx}(\cot^{-1} x) = -\frac{1}{x^2+1}$$

$$\circ \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}} \quad \Rightarrow \quad \int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + C$$

$$\circ \frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

Note: the formula for the derivative of the inverse secant assumes a restricted domain for the secant of $\{x \mid 0 < x < \pi/2 \text{ or } \pi < x < 3\pi/2\}$. If instead the restricted domain is

$(0, \pi/2) \cup (\pi/2, \pi)$, then the differentiation rule becomes $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$.

6.6c

Exercises:

Find the derivative of the function. Simplify where possible.

- $g(x) = \arccos \sqrt{x}$
- $y = \cos^{-1}(\sin^{-1} t)$
- $R(t) = \arcsin(1/t)$

6.6d

$$\bullet y = \arctan \sqrt{\frac{1-x}{1+x}}$$

6.6e

Evaluate the integral

- $\int_{-1/\sqrt{2}}^{1/\sqrt{2}} \frac{6}{\sqrt{1-p^2}} dp$
- $\int_0^{\sqrt{3}/4} \frac{dx}{1+16x^2}$